Per-subcarrier Antenna Selection for OFDMA-based Cognitive Radio Systems

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***Abstract*— The performance of an OFDM system can be improved by performing per-subcarrier antenna selection, which facilitates the exploitation of frequency and spatial diversity in the wireless channel. In this paper, we extend the concept of per-subcarrier antenna selection to a multiuser cognitive radio environment and present a practical subcarrier and antenna selection algorithm that exploits the multiuser, frequency and spatial diversities inherent in such systems while requiring only limited channel knowledge. The problem is formulated as an integer programming (IP) problem. We demonstrate that a linear relaxation of the problem still leads to an optimal solution, thus reducing the computational complexity relative to other approaches found in the literature. Simulation results demonstrate that the proposed resource allocation problem leads to an improvement in secondary users' link qualities, compared to a single-input single-output system and, at the same time, limits the interference to the primary user.**

I. Introduction

With the rapid increase in the number of wireless devices and services over the last few years, it has become apparent that the wireless spectrum might not be able to support the future demands given the current allocation of spectrum. However, a recent study performed by the Federal Communications Commission (FCC) has demonstrated that large portions of licensed spectrum are not properly utilised [1]. To deal with the issue of inefficient spectrum usage, and consequently to solve the problem of frequency scarcity, cognitive radio (CR) [2] has been proposed where unlicensed secondary users (SUs) can opportunistically access a primary network. The performance of such a system is, however, limited by the interference power constraint (IPC), which is typically imposed to protect licensed users.

Finding the most appropriate resource allocation strategy for SUs given the limitations of the primary network has been an area of active study over the last couple of years (see, e.g., [3], [4] and references therein). Given the potential capacity benefits that multiple-input multiple-output (MIMO) systems can provide without requiring extra power or bandwidth, the possible applications of MIMO in CR environments has incited interest in the research community [5], [6]. Cognitive MIMO systems can improve the link quality of SUs while limiting, or completely removing, the interference to the primary system through beamforming techniques. Unfortunately, these benefits come at the cost of an increase in the hardware and computational complexity and usually require full knowledge of the channel state information (CSI) at the transmitter side. As such, these approaches cannot always be readily deployed in practice.

One way of dealing with the high cost of a MIMO system, while retaining most of its benefits, is to perform antenna selection, where only a subset of the antennas is used for transmission and/or reception. Recent work on antenna selection in CR networks includes [7], where the authors investigate different criteria for antenna selection in a downlink, multiple-input single-output (MISO) system and show that the rate of the cognitive user can be increased while still satisfying the interference constraint of the primary user (PU). In [8], on the other hand, a MIMO system with receive antenna and user selection in a multiuser cognitive radio is presented where it is shown that the throughput of the cognitive radio system can be improved.

In contrast to the narrowband systems considered in the literature, in this paper, we present a resource allocation algorithm for the downlink transmission in a multicarrier, multiuser, MISO cognitive environment. Assuming multiple SUs share the spectrum with the PU, we propose a novel subcarrier and antenna selection algorithm that can be solved optimally using common linear programming solvers. The motivation behind this work is to present an optimal, low complexity resource allocation technique that can be readily deployed in dynamic cognitive scenarios. To this end, the set of constraints in the proposed optimisation problem does not explicitly define the IPC of the PU. Nevertheless, by an appropriate choice of the objective function, it is demonstrated that it is still possible to stay below the IPC threshold with a high probability, and simultaneously benefit from diversity gains. Moreover, the antenna selection process presented herein only requires knowledge of the channel gains, which can be obtained efficiently through a limited feedback channel or, in time division duplex (TDD) systems, by exploiting channel reciprocity.

This paper is organised as follows. The system model is given in section II. The proposed problem formulation for transmit antenna and subcarrier selection is given in section III. Section IV presents two objective functions that can be used in the optimisation problem. Simulation results and discussion are given in section V, and section VI provides some concluding remarks.

Notation: We use bold uppercase (lowercase) letters to represent matrices (vectors); $\mathbf{1}_M$ is a vector of 1's of dimension $M \times 1$ and $\mathbf{0}_{M\times N}$ is the all zero matrix of dimension $M \times N$; $\otimes$ represents the Kronecker product and $(\cdot)^T$ denotes the transpose operation.

II. System Model

We consider a centralised multiuser, cognitive MISO-OFDM system with M transmit antennas sharing the same spectrum as a PU, as depicted in Fig. 1. Note that although a single PU with a single receive antenna is considered in this paper, the technique presented can be readily extended to other cases.

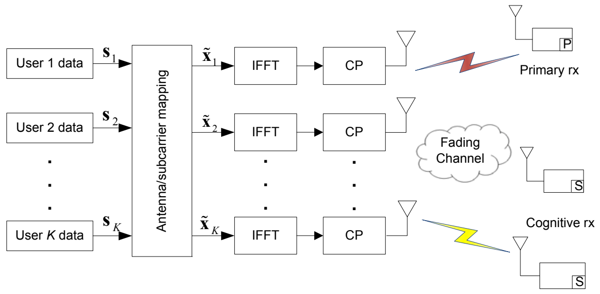

Fig. 1. Block Diagram of MISO CR system

Assuming a total of N subcarriers and K SUs, the aim of this system is to allocate subcarriers among the SUs and assign at most one antenna to each subcarrier. If n_k subcarriers are assigned to user k, the user data to subcarrier mapping process can be represented as

$$\mathbf{X}_k = \mathbf{P}_k \mathbf{s}_k \tag{1}$$

where $\mathbf{s}_k$ is the $n_k \times 1$ symbol vector of user k and $\mathbf{P}_k$ is the $N \times n_k$ sparse binary matrix that maps the user's symbols on the correct subcarriers. If $\mathcal{N}_k$ is the set containing the indexes of subcarriers assigned to user k, then $\mathbf{P}_k$ is obtained by replacing the row of $\mathbf{0}_{N\times n_k}$, indexed by the ith entry of $\mathcal{N}_k$, by the ith row of an $n_k \times n_k$ identity matrix for $i = 1, \cdots, |\mathcal{N}_k|$. In this paper, we make the practical assumption that no subcarrier is shared among users, i.e., $\mathcal{N}_k \cap \mathcal{N}_{k'} = \emptyset, \forall k \neq k'$.

The process of mapping the resulting loaded subcarriers to different antennas can be similarly expressed as

$$\bar{\mathbf{X}}_m = \mathbf{Q}_m \tilde{\mathbf{X}}, \qquad \bar{\mathbf{X}}_m \in \mathbb{C}^{N\times 1} \tag{2}$$

where $\tilde{\mathbf{X}} = \sum_{k=1}^{K} \mathbf{X}_k$ and $\mathbf{Q}_m$ is the $N \times N$ sparse, diagonal matrix that selects the carriers assigned to the mth antenna for transmission. The ith diagonal entry of $\mathbf{Q}_m$ is equal to one only if the ith subcarrier is mapped to the mth antenna.

The signal transmitted from the mth antenna at the cognitive base station, prior to adding the cyclic prefix, can be expressed as

$$\mathbf{x}_m = \mathbf{F}^H \bar{\mathbf{X}}_m \tag{3}$$

where $\mathbf{F}$ is the $N \times N$ Fourier transform matrix given by $[\mathbf{F}]_{m,n} = 1/\sqrt{N} \exp(-j2\pi(m-1)(n-1)/N)$.

III. Antenna Selection

The purpose of the per-subcarrier antenna selection in a multiuser cognitive radio environment is to assign the most adequate subcarriers and antennas to each SU. With such a selection, it is possible to exploit multiuser, frequency and spatial diversity to improve the signal quality of the cognitive users while limiting the interference to the licensed user.

We assume that the cognitive transmitter has knowledge of the channel gains between the cognitive transmitter and the cognitive receivers as well as between the cognitive transmitter and the primary receiver (termed as the interference channel). The latter assumption holds provided there is some cooperation between the primary and secondary system or, in the case of TDD links, channel reciprocity is used. Let the channel between the secondary transmitter and cognitive user k be expressed as

$$\mathbf{H}_k = [\tilde{\mathbf{h}}_1^k, \tilde{\mathbf{h}}_2^k, \cdots, \tilde{\mathbf{h}}_M^k] \tag{4}$$

where $\tilde{\mathbf{h}}_m^k = [\tilde{h}_{1,m}^k, \tilde{h}_{2,m}^k, \cdots, \tilde{h}_{N,m}^k]^T$ is the frequency response of the channel of user k from the mth transmit antenna. Similarly, the interference channel between the secondary transmitter and the primary receiver is expressed as

$$\mathbf{G} = [\tilde{\mathbf{g}}_1, \tilde{\mathbf{g}}_2, \cdots, \tilde{\mathbf{g}}_M] \tag{5}$$

where $\tilde{\mathbf{g}}_m = [\tilde{g}_{1,m}, \tilde{g}_{2,m}, \cdots \tilde{g}_{N,m}]^T$. In this paper, we assume that all channels are independent and identically distributed (*i.i.d*).

A. Optimisation Problem

We formulate the resource allocation for the cognitive MISO system as

$$\begin{aligned} \text{maximise} \quad & f(\mathbf{H}_1, \cdots, \mathbf{H}_k, \mathbf{G}, \rho) \\ \text{subject to} \quad & \sum_{n=1}^{N} \sum_{m=1}^{M} \rho_{k,n,m} \geq n_{min}, \quad \forall k \\ & \sum_{k=1}^{K} \sum_{m=1}^{M} \rho_{k,n,m} \leq 1, \quad \forall n \\ & \rho \in \{0, 1\} \end{aligned} \tag{6}$$

where $f(\mathbf{H}_1, \cdots, \mathbf{H}_k, \mathbf{G}, \rho)$ is a function of the gains of the SUs' channels and the interference channel as well as the indicator variables ρ. It will be shown in the next section how this objective function can be designed to find a compromise between the interference to the PU and the collective performance of the SUs. We define the indicator (optimisation) variable as

$$\rho_{k,n,m} = \begin{cases} 1, & \text{if subcarrier } n \text{ is assigned to user } k \\ & \text{on the } m\text{th antenna} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

The first constraint in (6) ensures some fairness among the SUs by stating that each user should be assigned a minimum number of subcarriers, n_{min}, while the second constraint ensures that no subcarrier is shared among users and that each subcarrier is assigned to at most one antenna. Note that in order for the problem to have a feasible solution, one must ensure that $n_{min} \leq \frac{N}{K}$.

In matrix notation, the integer programming (IP) problem in (6) can be expressed as

$$\begin{aligned} \text{maximise} \quad & \mathbf{c}^T \rho \\ \text{subject to} \quad & \mathbf{A}_1 \rho \leq \mathbf{1}_N \\ & \mathbf{A}_2 \rho \geq \mathbf{n} \end{aligned} \tag{8}$$

where $\mathbf{c}$ is the objective function and $\rho = [\rho_{1,1,1}, \rho_{1,1,2}, \cdots, \rho_{K,N,M}]^T$. The constraint matrices are then defined as

$$\mathbf{A}_1 \triangleq [\mathbf{1}_K \otimes \mathbf{I}_N \otimes \mathbf{1}_M]^T, \quad \mathbf{A}_2 \triangleq [\mathbf{I}_K \otimes (\mathbf{1}_{NM})^T] \tag{9}$$

B. Linear Relaxation

Note that the feasible set of the integer programming (IP) problem in (8) is given by

$$\mathcal{F} = \left\{ \rho \in \{0,1\}^{KNM} \middle| \mathbf{A}_1 \rho \leq \mathbf{1}_N, -\mathbf{A}_2 \rho \leq -\mathbf{n} \right\} \tag{10}$$

where $\mathbf{n} = \mathbf{1}_K \times n_{min}$. A linear relaxation of the above would lead to

$$\mathcal{P} = \left\{ \begin{array}{l} \rho \in \mathbb{R}^{KNM} \middle| \mathbf{A}_1 \rho \leq \mathbf{1}_N, -\mathbf{A}_2 \rho \leq \mathbf{n}, \\ \mathbf{0}_{KNM \times 1} \leq \rho \leq \mathbf{1}_{KNM} \end{array} \right\} \tag{11}$$

Similar to the work presented in [9], it can be verified that the feasible set of the relaxed problem is totally unimodular[1] [10]. Consequently, provided the objective function is affine in ρ, running linear programming algorithms, such as the simplex or the interior point method, would produce the desired integer results at a greatly reduced complexity.

The problem formulation in this paper differs from existing work in that there is no explicit constraint to limit the aggregate interference to the PU to a given threshold [11], [12]. The latter constraint is often defined as

$$\sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{M} |\tilde{g}_{n,m}|^2 \rho_{k,n,m} p_s \leq \mathcal{I}_{th} \tag{12}$$

where $\mathcal{I}_{th}$ is the maximum allowable interference and p_s is the transmit power on each subcarrier. The reason for not including this constraint is to avoid destroying the total unimodularity of the constraint matrix. If the constraint was employed, the optimal solution would be obtained by running integer program solvers, such as the *branch-and-bound* method [10], which would greatly increase the computational complexity. Such a situation is to be avoided in a dynamic cognitive radio environment. Simulation results in section V will, however, demonstrate that the interference to the primary system can be controlled by a careful choice of the objective function.

IV. Choice of Objective Function

The objective function in (6) is a function of the gains of the SUs' channels and the interference channel, as well as the indicator variables. We consider two objective functions that can be used in this system [13], [14].

[1]A matrix $\mathbf{A}$ is totally unimodular if every square submatrix of $\mathbf{A}$ has determinant 0, 1 or -1.

A. Ratio of Channel Gains

The first objective function considered is

$$f(\mathbf{H}_1, \cdots, \mathbf{H}_k, \mathbf{G}, \rho) = \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{M} \rho_{k,n,m} \frac{|\tilde{h}_{n,m}^k|^2}{|\tilde{g}_{n,m}|^2}. \tag{13}$$

With such a definition, the interference channel gains and the secondary link gains can be viewed as the cost and benefit, respectively, for transmitting on a particular subcarrier from a given antenna. The optimisation process will assign those subcarriers and antennas to each SU that has the highest channel gain ratios. This corresponds to selecting the antenna/subcarrier pairs that have the highest secondary link gain and lowest interference channel gain, and so reduce the interference to the PU.

B. Weighted Difference of Channel Gains

Although taking the ratio of the channel gains provides diversity gain and will reduce the interference to the PU as the number of transmit antennas increases, the formulation does not provide adequate control over the interference level. For this reason, we propose a second objective function, which is defined as

$$\begin{aligned} f(\mathbf{H}_1, \cdots, \mathbf{H}_k, \mathbf{G}, \rho) = \delta \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{M} |\tilde{h}_{n,m}^k|^2 \rho_{k,n,m} \\ -(1-\delta) \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{m=1}^{m} |\tilde{g}_{n,m}|^2 \rho_{k,n,m}. \end{aligned} \tag{14}$$

The variable $\delta \in [0, 1]$ in (14) is included to provide a trade-off between the interference observed by the primary receiver and the sum link quality of the SUs. When $\delta = 0$, the objective function is equivalent to minimising the interference channel under fixed power loading at the secondary base station. On the other hand, when $\delta = 1$, the objective function is equivalent to maximising the sum channel gain of the secondary links and consequently reducing error probabilities and increasing capacity. By choosing the right value for δ, one is able to achieve an adequate trade-off between the interference to the primary receiver and the secondary link quality. Note that the choice of the objective function does not affect the unimodularity properties of the constraint matrix.

V. Simulation Results

In this section, we analyse the performance of the proposed antenna selection strategy. We start by investigating the effect of the parameter δ on the aggregate interference to the PU and the sum channel gain of the SUs. Based on the IP formulation presented above, the aggregate interference, assuming equal power allocation on all subcarriers, is calculated using the left hand side of (12). We consider all channels to be exponentially decaying with 8 channel taps and a decay factor of 0.86.

Consider a system with $N = 128$ subcarriers, $K = 4$ SUs, $n_{min} = 30$, one PU receiver listening to all N subcarriers and assume that the transmit power on each subcarrier is normalised to one, i.e. 0 dB. Fig. 2 shows the interference observed by the PU for different values of δ (averaged over 2000 channel realisations) and different numbers of antennas at the cognitive transmitter. It can be observed that for values of $\delta < 1$, increasing the number of antennas results in a decrease in interference power. Such a behaviour can be attributed to the extra degree of freedom provided by multiple transmit antennas. At $\delta = 1$, regardless of the number of transmit antennas, the amount of interference converges to the maximum of N. This can be explained by the structure of the objective function in the optimisation problem. At $\delta = 1$, the objective function is solely to maximise the sum channel gain of the SU without any consideration to the PU's performance. Similarly, at $\delta = 0$, the goal of the optimisation is to reduce the interference to the licensed user, subject to each SU being assigned 30 subcarriers. In that case, the aggregate channel gain of the SU, shown in Fig. 3, does not depend on the number of transmit antennas.

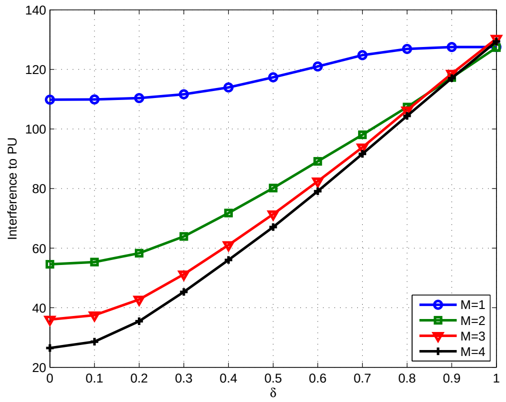

Fig. 2. Aggregate interference to PU with $N = 128$, $K = 4$, $n_{min} = 30$ and p_s=1

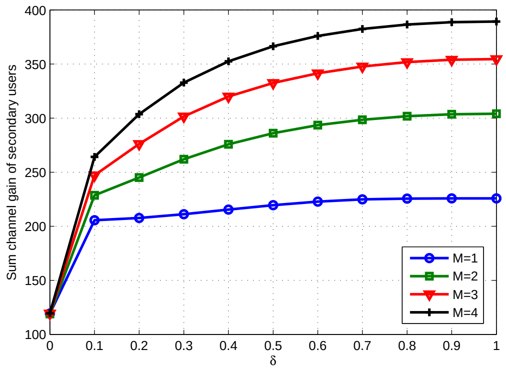

Fig. 3. Aggregate gain of SUs with $N = 128$, $K = 4$, $n_{min} = 30$ and p_s=1

In Fig. 4, the bit error rate (BER) plot for the system with 128 subcarriers and 4 secondary users is shown for 3 different values of δ with two transmit antennas. As made clear from the two previous figures, changing the value of the parameter δ causes a change in the performance of the cognitive users.

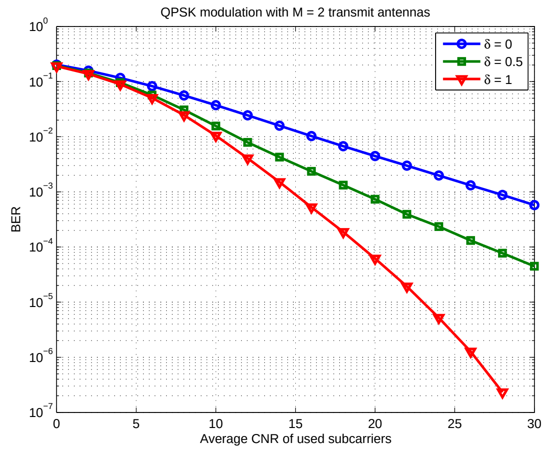

Fig. 4. BER plot for a system with 2 transmit antenna and QPSK modulation

To show the benefits of performing transmit antenna selection at the cognitive base station, we compare the proposed system with the IP based resource allocation presented in [11]. In that paper, the authors formulate the subcarrier and bit loading operation as an IP and propose to use the branch-and-bound method to obtain the optimal solution, subject to a total IPC. To compare that method with the system proposed in this paper, we assume that no bit/power loading is performed in the allocation algorithm of [11], as this would further increase the complexity, i.e., the algorithm performs only subcarrier allocation, and we replace the objective function of maximising the aggregate rate by the function of maximising the sum channel gain. This modified algorithm is taken as the benchmark. Fig. 5 presents the complementary cumulative distribution function (ccdf) of the total interference to the PU for our proposed system and the modified version of [11]. Under both schemes, a fixed power allocation on each of the $N = 64$ subcarriers is considered in a $K = 4$ SUs system with $n_{min} = 15$. The IPC was set to 40 in the benchmark. We consider systems with $M = 3$ and $M = 4$ transmit antennas with $\delta = 0$ and $\delta = 0.3$.

Because of the explicit interference constraint in the benchmark, the sum interference with the latter algorithm is upper bounded by the IPC value. It can be observed from the figure that increasing the number of antennas while keeping the value of δ fixed leads to a decrease in the interference power. Also, when the ratio of channel gains is taken as the objective function and $M = 4$, the probability of exceeding the IPC is similar to that when the weighted difference of channel gain is taken as objective function and $\delta = 0.3$. Nevertheless, the only means of controlling the interference to the PU and sum channel gain to the SUs is by increasing the number of transmit antennas when ratio selection is employed. With $\delta = 0$ values, the probability of exceeding the interference threshold is greatly reduced. However, with $\delta = 0$, the optimisation problem is identical to a minimisation of the interference

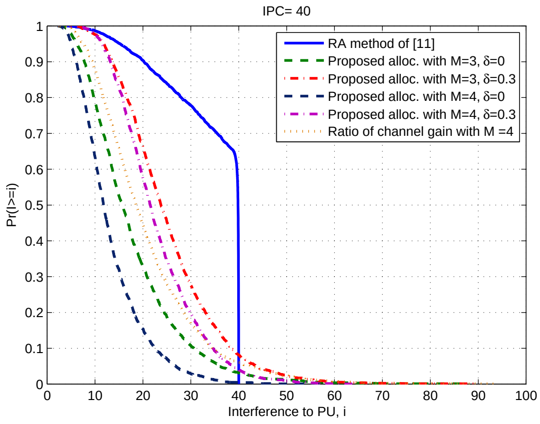

Fig. 5. ccdf of interference power to PU with $N = 64$ and $K = 4$ SUs

power, disregarding the secondary link. This point is brought out in Fig. 6, which shows the ccdf of the sum channel gains under the same simulation parameters as above.

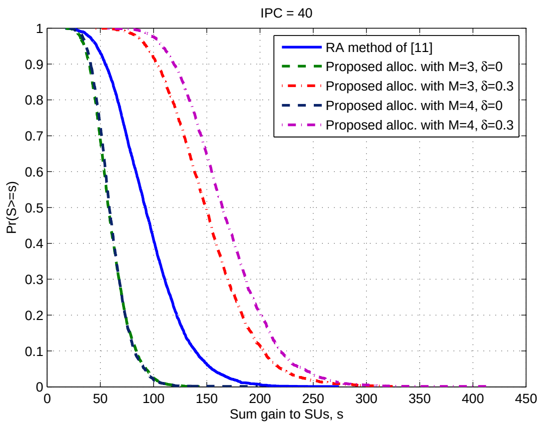

Fig. 6. ccdf of gain to SUs with $N = 64$ and $K = 4$ SUs

It can be observed here that $\delta = 0$ yields poorer performance, irrespective of the number of transmit antennas. However, for $\delta = 0.3$, a significant increase in the sum channel gain can be seen with increasing numbers of antennas. Thus, for a given non-zero δ value, it is possible to reduce the interference to the PU, and at the same time improve the link quality of the cognitive users by increasing the number of transmit antennas. Alternatively, decreasing the value of δ would reduce the interference at the expense of reducing the link quality of the secondary users.

It should be pointed out again here that although the benchmark strictly adheres to the IPC, obtaining the optimal solution requires solving a non-ideal IP which is highly complex. On the other hand, the method proposed in this paper can be efficiently solved through linear relaxation techniques, which makes it more practical. One drawback, however, is that the interference to the primary network can go beyond the threshold occasionally.

VI. Conclusion

In this paper, we have presented a means of allocating subcarriers among secondary users and assigning each subcarrier to a transmit antenna so that the interference to the primary user can be controlled while the link quality of the secondary users is improved. It was shown through simulations that the solution presented in this paper can produce results very close to a system with an explicit interference power constraint, by either increasing the number of transmit antennas or decreasing the value of the parameter δ. Moreover, given the structure of the problem formulation, the optimal solution can be obtained using common linear programming solvers making it ideal for practical systems.

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